2.7 Solving Problems Involving More than One Right Triangle

We can use trigonometry to solve problems that can be modelled using right triangles. When more than one right triangle is involved, we have to decide which triangle to start with.

Calculate Unknown Side Length Using More than One Triangle

Example 1. Calculate the length of CD to the nearest tenth of a centimetre.

\[ \sin 47^\circ = \frac{4.2}{BD} \]

\[ BD = \frac{4.2}{\sin 47^\circ} = 5.74 \text{ cm} \]

\[ \cos 26^\circ = \frac{CD}{5.74} \]

\[ CD = 5.74 \cdot \cos 26^\circ \]

\[ CD = 5.2 \text{ cm} \]

Example 2. Determine the length of PQ, to the nearest 0.1 cm.

\[ \tan 31^\circ = \frac{PR}{6.3} \]

\[ PR = 6.3 \cdot \tan 31^\circ \]

\[ PR = 3.785 \text{ cm} \]

\[ \tan 43^\circ = \frac{QR}{6.3} \]

\[ QR = 6.3 \cdot \tan 43^\circ \]

\[ QR = 5.825 \text{ cm} \]

\[ \text{Find PQ} = \text{QR} - \text{PR} = 2.1 \text{ cm} \]

Try. In the diagram QS = 32 mm, \( \angle PQS = 50^\circ \) and \( \angle RPS = 61^\circ \). Determine the RS to the nearest mm.

\[ \tan 50^\circ = \frac{PS}{32} \]

\[ PS = 32 \cdot \tan 50^\circ \]

\[ PS = 38.136 \text{ mm} \]

\[ \tan 61^\circ = \frac{RS}{38.136} \]

\[ RS = 38.136 \cdot \tan 61^\circ \]

\[ RS = 69 \text{ mm} \]
Calculate Unknown Angle Measure Using More than One Triangle

Example 3. Calculate the measure of \( \angle \text{BEC} \) to the nearest degree.

\[ \begin{align*}
\text{First, find } & \quad \text{EC} \\
& \quad \sin 39^\circ = \frac{\text{EC}}{3.7} \\
& \quad \text{EC} = 3.7 \cdot \sin 39^\circ \\
& \quad \text{EC} = 2.328 \text{ m} \\
\end{align*} \]

\[ \begin{align*}
\text{Then, find } & \quad \angle \text{BEC} \\
& \quad \cos \theta = \frac{2.328}{4.6} \\
& \quad \theta = \cos^{-1} \left( \frac{2.328}{4.6} \right) \\
& \quad \theta = 60^\circ \\
\end{align*} \]

Example 4. Calculate the measure of \( \angle \text{ABC} \) to the nearest degree.

\[ \begin{align*}
\text{First, find } & \quad \angle \text{DAC} \\
& \quad \tan \angle \text{DAC} = \frac{4.9}{7.4} \\
& \quad \angle \text{DAC} = \tan^{-1} \left( \frac{4.9}{7.4} \right) \\
& \quad \angle \text{DAC} = 33.51^\circ \\
\end{align*} \]

So \( \angle \text{ABC} = 180^\circ - 46.51^\circ - 90^\circ = 43.49^\circ = 43^\circ \)

Try. Calculate the measure of \( \angle \text{ABD} \) to the nearest degree.

\[ \begin{align*}
\text{First, find } & \quad \text{BD} \\
& \quad \cos 38^\circ = \frac{\text{BD}}{24.3} \\
& \quad \text{BD} = 19.1486 \\
\end{align*} \]

\[ \begin{align*}
\text{Then, find } & \quad \angle \text{ABD} \\
& \quad \cos \angle \text{ABD} = \frac{19.1486}{19.9} \\
& \quad \angle \text{ABD} = 16^\circ \\
\end{align*} \]
The **angle of depression** of an object below the horizontal is the angle between the horizontal and the line of sight from an observer.

**Solve a Problem with Triangles in the Same Plane**

Example 5. From the top of a 20-m high building, a surveyor measured the angle of elevation of the top of another building and the angle of depression of the base of that building. The surveyor sketched this plan of her measurements. Determine the height of the taller building to the nearest tenth of a metre.

In order to find AC, we will find DB, followed by AD.

1. Find DB
   \[
   \tan 15^\circ = \frac{20}{DB} \quad \therefore \quad DB = 74.64\ m
   \]

2. Find AD
   \[
   \tan 30^\circ = \frac{AD}{74.64} \quad \therefore \quad AD = 43.09\ m
   \]

3. Find AC = AD + DC
   \[
   AC = 63.09 + 20 = 63.1\ m
   \]

Try. In January 2003, the tallest building in Rockville was the Metro Building. Recently, a developer was commissioned by the Gammapro Oil Company to build a taller building next to the Metro Building. From the top of the Metro Building, the angle of elevation of the top of the Gammapro Building is 24° and the angle of depression to the foot of the Gammapro Building is 56°. If the buildings are 45 m apart, determine the height of each building to the nearest metre.

1. Find \( x \)
   \[
   \tan 56^\circ = \frac{x}{45} \quad \therefore \quad x = 45 \cdot \tan 56^\circ = 66.715\ m
   \]

2. Find \( y \)
   \[
   \tan 24^\circ = \frac{y}{45} \quad \therefore \quad y = 45 \cdot \tan 24^\circ = 20.035\ m
   \]

Height of Gammapro = \( x + y \)

\[
= 66.715 + 20.035 = 86.75\ m
\]

Height of Metro = 67 m
Solve a Problem with Triangles in the Different Planes

Example 6. From the top of a 90-ft. observation tower, a fire ranger observes one fire due west of the tower at an angle of depression of 5°, and another fire due south of the tower at an angle of depression of 2°. How far apart are the fires to the nearest foot? (textbook p.116)

Try. Ann and Byron positioned themselves 35 m apart on one side of a stream. Ann measured the angles, as shown below. Calculate the height of the cliff on the other side of the stream.