Lesson 3: Pathways

Pure Math 30: Explained!

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Pascal's Triangle: The following triangle is used in solving pathway problems, so you will need to learn the pattern.

- Start the triangle at the top with the number 1.
- Slide the 1 down diagonally to form the beginning & ends of each row.
- To fill in the rows, add together the numbers immediately above. (Diagram)

Note that Pascal's Triangle is made up entirely of combinations!

- The first row is: $\binom{0}{0}$
- The second row is: $\binom{1}{0}, \binom{1}{1}$
- The third row is: $\binom{2}{0}, \binom{2}{1}, \binom{2}{2}$

This pattern continues forever.
**Permutations & Combinations**

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**Simple Pathways:** You can use the principles of Pascal's Triangle to find out how many possible pathways exist from one point in a grid to another point in the grid.

**Example 1:** How many paths exist from point A to B?

Step 1: First slide 1’s across the outer edge to form the edge of Pascal’s Triangle.

Steps 2 through 6: Fill in the Pascal Triangle Pattern.

Step 3: Fill in more spaces

Step 4: Fill in more spaces

Step 5: Fill in more spaces

Step 6: The final number in the pattern is your answer.

There are 20 possible paths.

**Example 1 Shortcut:**
Note that you can go East 3 times, and Down 3 times. Writing out the letters gives you EEEDDD. How many ways can you arrange these letters? You can do it in:

\[
\frac{6!}{3! \times 3!} = 20 \text{ ways.}
\]

This shortcut will only work when there are no gaps or extra spaces in the pathway.

**Example 2:** In the following cube, how many paths exist from point A to B?

In the 3-D cube, we can write out all the possible directions we can go. We can move East 4 times, Down 5 times, and Forward 3 times. Writing out the letters gives you EEEEDDDDDFF.

You can do it in:

\[
\frac{12!}{4! \times 5! \times 3!} = 27720 \text{ ways.}
\]
**Complex Pathways:** More difficult pathway problems involve gaps & spaces in the grid, so you have to be careful when applying Pascal’s triangle.

**Example 3:** Find the number of paths from point A to B:

**Step 1:** First slide 1’s across the outer edge to form the edge of Pascal’s Triangle.

**Step 2:** Now fill in as many points as you can.

**Step 3:** Slide 35’s across the second rectangle.

**Step 4:** Fill in the second rectangle.

You could also do this question by considering these to be two separate rectangles and multiplying the answers together. From the first one, we have $\text{EEEESSS} = \frac{7!}{4! \cdot 3!} = 35$.

From the second, we have $\text{EESS} = \frac{4!}{2! \cdot 2!} = 6$. Multiplying these results, we get $35 \times 6 = 210$. 

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Example 4: Find the number of paths from point A to B:

Step 1: First slide 1's across the outer edge to form the edge of Pascal's Triangle.

Step 2: Now fill in as many points as you can.

Step 3: Slide 6's across the top wall.

Step 4: Now fill in as many points as you can.

Step 5: Slide 52's down the left wall.

Step 6: Fill in the remaining points. There are 552 paths.
Example 5: Find the number of paths from point A to B:

Step 1: First slide 1's across the outer edge.

Step 2: Now fill in as many points as you can.

Step 3: Since there is a gap in the grid, slide the 6 over to the next space.

Step 4: Fill in as many points as you can.

Step 5: Slide the numbers along the edge to continue the pattern.

Step 6: Fill in the remaining points.

There are 165 paths.
Example 6: Find the number of paths from point A to B:

**Step 1:**
First complete the points required to get to the dot.

**Step 2:**
Slide 4's right & down.

**Step 3:**
Fill in the remaining points. There are 16 paths.

Example 7: Find the number of paths from point A to B:

**Step 1:**
First place the 1's to start off Pascal's Triangle.

**Step 2:**
There are two possible paths, as indicated by the arrows.

**Step 3:**
There are three paths in total to get to the middle point.

**Step 5:**
Slide the 3's to find the remaining paths.

**Step 6:**
There are a total of six paths to B.
Example 8: A pinball game has a series of pegs which create multiple paths for a ball to reach the bottom. How many paths lead to $X$?

Step 1:
Use the pegs to create Pascal's triangle.

Interesting Variation:
In Example 8, the pegs are used to form Pascal's triangle since the $X$ can be reached diagonally from the two pegs immediately above it.

If the $X$ can't be reached diagonally from two pegs immediately above it, the spaces between the pegs must be used to find the number of pathways.

Example:

There are 4 ways the ball can reach $X$. 
Questions: Find the number of paths from A to B in each of the following:

1) A
2) A
3) A
4) A
5) A
6) A
7) A
8) A
For each of the following, find the number of paths through the dot.

9) A

10) A

11) A

12) A

13) A

For each of the following, find the number of paths leading to X.

14) 

15) 

16) 

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For each of the following, find the number of paths from left to right.

17) [Pathway Diagram]

18) [Pathway Diagram]

19) [Pathway Diagram]

20) [Pathway Diagram]

Answers:

1. [Pathway Diagram]
2. [Pathway Diagram]
3. [Pathway Diagram]
4. [Pathway Diagram]
5. [Pathway Diagram]
6. [Pathway Diagram]
7. [Pathway Diagram]
8. [Pathway Diagram]
Permutations & Combinations

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9. FFFFFEEDDDD = $\frac{13!}{5! \cdot 4! \cdot 4!} = 90090$

10. FFFEEEEDDDD = $\frac{11!}{3! \cdot 4! \cdot 4!} = 11550$

11. A

12. A

13. A

14. B

15. B

16. B

17. 6

18. 6

19. 4

20. 7